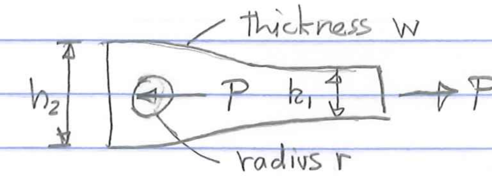
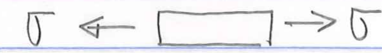
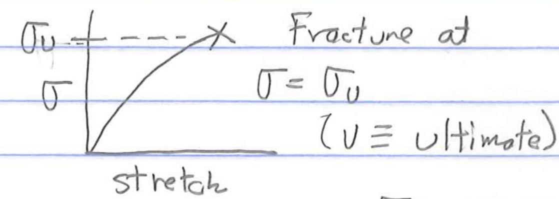
3. Design concepts

Consider the bar in the previous example



Suppose the material is very brittle:

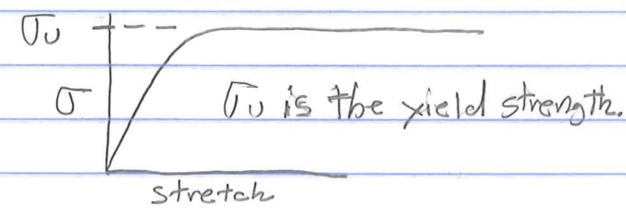




Then we would base the design of the bar on $\sigma_{max} \leq \frac{\sigma_U}{F.S.}$, where F.S. = factor of safety (>1) and σ_{max} is the maximum value of stress σ in the bar produced by the load P. Note that this requires an accurate stress analysis be performed on the bar. This method is called allowable stress design:

$$\sigma_{max} \leq \sigma_{allow} \quad \text{where } \sigma_{allow} = \sigma_U / F.S.$$

Suppose the material is ductile:



Then the bar will not fail until an entire cross-section has reached yield. This load is $P_U = \sigma_U \cdot A_{min}$ where

$$A_{min} = \min(h_1 w, (h_2 - 2r) w).$$

Thus, design can be based on

$$P \leq P_U / F.S.$$

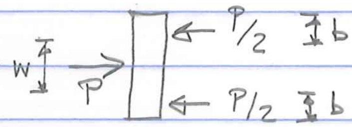
No accurate stress analysis is required. This method is called strength design.

A "pseudo" allowable stress design, equivalent to strength design and likewise appropriate for ductile material, can be expressed as follows

$$(\sigma_{av})_{max} \leq \sigma_{allow} \quad \text{where } \sigma_{allow} = \sigma_U / F.S. \text{ and}$$

where $(\sigma_{av})_{max}$ is the maximum of the average stress (over all cross-sections) $= P/A_{min}$.

Now consider the pin:



The same concepts apply. For the "pseudo" allowable stress design for ductile material, the equations are:

For shear: $(\tau_{av})_{max} \le \tau_{allow}$ where $\tau_{allow} = \tau_U / F.S.$

For the example: $(\tau_{av})_{max} = P/2 \,/\, \pi R^2$

For bearing: $(\sigma_{b,av})_{max} \le \sigma_{b,allow}$ where $\sigma_{b,allow} = \sigma_{b,u} / F.S.$

For the example: $(\sigma_{b,av})_{max} = P / A_{min}$

where $A_{min} = \min(2wr, 4br)$

The average stresses used above in strength or "pseudo" allowable stress design are appropriate for axial forces in bars, shear forces in pins, and bearing forces in pin connections. Other cases, such as bending and twisting, require more refined stress analysis, as done later in the term.

An improved version of strength design using a more rational version of the factor of safety can be expressed as follows:

$$F.S. = \gamma \cdot \frac{1}{\phi}$$

where γ = load factor and ϕ = resistance factor, where γ (>1) accounts for uncertainty in loads and ϕ (<1) accounts for uncertainty in the material.

Using the example of axial force in a bar,

$$P \le P_U / F.S.$$

or $\quad \gamma P \le \phi P_U$.

Further, we can use different values of γ for contributions to P from different sources:

$$\gamma_D P_D + \gamma_L P_L \le \phi P_U$$

where $D \equiv$ dead load and $L \equiv$ live load. Typically, $\gamma_L > \gamma_D$ since more uncertainty would be associated with the live load contribution.